MCQ'S – 3; Two Marks – 2; FB – 1; 2 Marks – 2; 3 Marks – 2; 5 Marks – 1

MCQ'S:

1. A unit vector parallel to the sum of the vectors 2i + 3j – k and 4i + 2j + k is
   a. $\frac{6i+5j}{\sqrt{61}}$ b. $\frac{5i+6j}{\sqrt{61}}$ c. k d. none of these

2. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ then
   a. $\vec{a}$ and $\vec{b}$ are perpendicular c. $|\vec{a}|=|\vec{b}|$
   b. $\vec{a}$ and $\vec{b}$ are parallel d. there is no relationship between $\vec{a}$ and $\vec{b}$

3. Given $\vec{a}$ = i + j – k, $\vec{b}$ = -i + 2j + k and $\vec{c}$ = -i + 2j – k, a unit vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is
   a. i b. j c. k d. $\frac{i+j+k}{\sqrt{3}}$

4. The direction cosiness of the vector, 3i – 4j + 5k are
   a. $\frac{3}{\sqrt{2}},\frac{-4}{\sqrt{2}},\frac{1}{\sqrt{2}}$ b. $\frac{3}{5},-\frac{4}{5},\frac{1}{5}$ c. $\frac{3}{5\sqrt{2}},\frac{-4}{5\sqrt{2}},\frac{1}{\sqrt{2}}$ d. $\frac{3}{5\sqrt{2}},\frac{4}{5\sqrt{2}},\frac{1}{\sqrt{2}}$

5. The area of the triangle two of whose sides are given by 4i – j + k and 3i + j – k is
   a. $7\sqrt{2}$ b. $14\sqrt{2}$ c. $\frac{14}{\sqrt{2}}$ d. $\frac{7}{\sqrt{2}}$

6. The adjacent sides of a parallelogram are i + 2j + 3k and 2i – j + k. its area is
   a. $3\sqrt{5}$ b. $5\sqrt{3}$ c. $\sqrt{15}$ d. $\frac{5\sqrt{3}}{2}$

7. The projection of $\vec{a}=3i+2k$ on the vector $\vec{b}=2i+3j+k$ is,
   a. $\frac{8}{\sqrt{35}}$ b. $\frac{8}{\sqrt{39}}$ c. $\frac{8}{\sqrt{14}}$ d. $\sqrt{14}$

8. The sine of the angle between the vectors 3i + j + 2k and i + j + 2k is
   a. $\sqrt{\frac{5}{22}}$ b. $\sqrt{\frac{5}{12}}$ c. $\sqrt{\frac{15}{22}}$ d. $\sqrt{\frac{5}{21}}$

9. If $|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2$, then
   a. $\vec{a}$ is parallel to $\vec{b}$ c. $\vec{a}=\vec{b}$
   b. $\vec{a}$ is perpendicular to $\vec{b}$ d. $|\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}|$

10. If θ is the angle between the vectors $\vec{a}$ and $\vec{b}$ then $\frac{|\vec{a}\times\vec{b}|}{\vec{a}\cdot\vec{b}}=$

a. $\cot\theta$ b. $-\cot\theta$ c. $\tan\theta$ d. $-\tan\theta$

11. For any vector $\vec{a}$, $(\vec{a}.i)i+(\vec{a}.j)j+(\vec{a}.k)k=$ (KCET 2004)

a. $\vec{a}$ b. $2\vec{a}$ c. $3\vec{a}$ d. $\vec{0}$

12. If $|\vec{a}|=5$, $|\vec{b}|=6$ and the angle between $\vec{a}$ and $\vec{b}$ is 60^0, then $\vec{a}.\vec{b}=$

a. 30 b. 15 c. $15\sqrt{3}$ d. $5\sqrt{3}$

13. If $|\vec{a} \times \vec{b}|=4$ and $|\vec{a}.\vec{b}|=2$ then $|\vec{a}|^2.|\vec{b}|^2=$

a. 6 b. 2 c. 20 d. 8

14. i.(j x k) + j. (k x i) + k. (i x j) =

a. 1 b. 3 c. -3 d. 0

(Kar CET 1994)

15. If any vector $\vec{a}$ $i \times (\vec{a} \times i)+j \times (\vec{a} \times j)+k \times (\vec{a} \times k)=$

a. i + j + k b. $3\vec{a}$ c. $2\vec{a}$ d. $\vec{a}$

16. The projection of $\vec{a}=5i-j+3k$ on $\vec{b}=2i+j-k$ is

a. 6 b. $\sqrt{6}$ c. $\sqrt{3}$ d. none of these

17. If $|\vec{a}|=5$, $|\vec{b}|=6$, $\vec{a}, \vec{b}=24$ then $|\vec{a} \times \vec{b}|=$

a. $\sqrt{224}$ b. 18 c. $\sqrt{300}$ d. $\sqrt{254}$

18. If $\vec{a}$ and $\vec{b}$ are unit vectors and $|\vec{a} \times \vec{b}|=1$ then the angle between $\vec{a}$ and $\vec{b}$ is

a. $\frac{\pi}{4}$ b. $\frac{\pi}{2}$ c. $\frac{\pi}{3}$ d. π

19. If θ is the angle between $\vec{a}$ and $\vec{b}$ and $|\vec{a} \times \vec{b}|=|\vec{a}.\vec{b}|$, the $\theta=$

a. 0 b. π c. $\frac{\pi}{2}$ d. $\frac{\pi}{4}$

20. Unit vector in the direction of $\vec{a}=2\hat{\imath}+3\hat{\jmath}+\hat{k}$ is

(a) $\frac{2\hat{\imath}+3\hat{\jmath}+\hat{k}}{14}$ b. $\frac{2\hat{\imath}-3\hat{\jmath}+\hat{k}}{14}$ c. $\frac{2\hat{\imath}+3\hat{\jmath}+\hat{k}}{\sqrt{14}}$ d. $\frac{2\hat{\imath}+3\hat{\jmath}-\hat{k}}{14}$

21. If α,β,γ are the angles that a line makes with the positive direction of x,y,z axis, respectively, then the direction cosines of the line are:

(a) $\sin\alpha,\sin\beta,\sin\gamma$ (b) $\cos\alpha,\cos\beta,\cos\gamma$ (c) $\tan\alpha,\tan\beta,\tan\gamma$ (d) $\cos^2\alpha,\cos^2\beta,\cos^2\gamma$

22. A line makes equal angles with co-ordinate axis. Direction cosines of this line are:

(a) $\pm(1,1,1)$ (b) $\pm\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ (c) $\pm\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ (d) $\pm\left(\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{-1}{\sqrt{3}}\right)$

23. The angle between the lines 2x = 3y =–z and 6x = –y = –4z is:

(a) 90° (b) 0° (c) 30° (d) 45°

24. The equation of straight line passing through the point (a, b, c) and parallel to z- axis, is:

(a) $\frac{x-a}{1}=\frac{y-b}{1}=\frac{z-c}{0}$ (b) $\frac{x-a}{0}=\frac{y-b}{1}=\frac{z-c}{1}$

(c) $\frac{x-a}{1}=\frac{y-b}{0}=\frac{z-c}{0}$ (d) $\frac{x-a}{0}=\frac{y-b}{0}=\frac{z-c}{1}$

25. The angle between two lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{-1}$ and $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-1}{2}$ is:

(a) $\cos^{-1}\left(\frac{4}{9}\right)$ (b) $\cos^{-1}\left(\frac{1}{3}\right)$ (c) $\cos^{-1}\left(\frac{2}{9}\right)$ (d) $\cos^{-1}\left(\frac{5}{9}\right)$

26. If a line makes angles $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with the x-axis and y-axis respectively. Then the angle made by the line with z-axis is

a. $\frac{\pi}{2}$ b. $\frac{\pi}{3}$ c. $\frac{\pi}{4}$ d. $\frac{5\pi}{12}$

27. If the direction cosine l, m, n of a line are 0, $\frac{1}{2}, \frac{\sqrt{3}}{2}$ then the angle made by the line with positive direction of y – axis is

(a) 60^0 (b) 30^0 (c) 90^0 (d) 45^0

28. If $\vec{a}$ is a non zero vector of magnitude 'a' and $\lambda\vec{a}$ is a unit vector, find the value of λ.

(a) a (b) $\frac{1}{a}$ (c) -a (d) $-\frac{1}{a}$

29. the direction ratios of the vector $\vec{a} = \hat{i} + \hat{j} - 2k$.

(a) (1, 1, 1) (b) (1, -1, 2) (c) (1, 1, -2) (d) (1, -2, 1)

30. The magnitude of the vector $\vec{a} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}k$ is

(a) 1 (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) – 1

31. The vectors $2\hat{i} - 3\hat{j} + 4k$ and $-4\hat{i} + 6\hat{j} - 8k$ are

(a) Co-planar vectors (b) collinear vectors

(c) perpendicular vectors (d) both are zero vectors

32. The vectors $3\hat{i} - 6\hat{j} + 2k$ and $6\hat{i} + 2\hat{j} - 3k$ are

(a) perpendicular to each other. (b) parallel to each other.

(b) Collinear vectors (d) coplanar vectors

33. If $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$, then what can you concluded about the vector $\vec{b}$?

(a) Zero vector (b) unit vector

(b) $\vec{b}$ (d) any vector

34. If the vectors $2\hat{i} + 3\hat{j} - 6k$ and $4\hat{i} - m\hat{j} - 12k$ are parallel, find 'm' is ……………..

35. The value of x, for which $x\left(\hat{i} + \hat{j} + k\right)$ is a unit vector is …………..

36. The value of $\hat{i} \cdot \left(\hat{j} \times k\right) + \hat{j} \cdot \left(\hat{i} \times k\right) + k \cdot \left(\hat{i} \times \hat{j}\right)$ is …………

37. If $\vec{a} = 2\hat{i} - \hat{j} + k$ and $\vec{b} = \hat{i} - 3\hat{j} - 3k$, then $\vec{a} \cdot \vec{b} =$ …………

38. If $\vec{a} = 2\hat{i} + \hat{j} + 3k$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2k$. $|\vec{a} \times \vec{b}| =$ ……………..

39. The value of λ, the vectors $\vec{a} = 2\hat{i} - 3\lambda\hat{j} + k$ and $\vec{b} = \hat{i} + \hat{j} - 2k$ are perpendicular to each other is …………………

40. The value of λ, for which vector $\frac{2}{3}\hat{i} - \lambda\hat{j} + \frac{2}{3}k$ is a unit vector is………………

41. If $\vec{a}$ is a unit vector such that $\left(\vec{x} - \vec{a}\right) \cdot \left(\vec{x} + \vec{a}\right) = 8$, find $\left|\vec{x}\right|$ =………………

FILL IN THE BLANKS:

42. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, the value of k is ....

43. A line makes acute angles α, β and γ with the coordinates axes such that cos α . cos β . cos $\gamma = \frac{2}{9}$ and cos γ cos α = $\frac{4}{9}$, then cos α + cos β + cos γ =................

44. If a line makes an angles α, β, γ with the coordinates axes then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is ....

45. If a line makes α , β , γ with the positive direction of x, y and z-axis respectively, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$ is ....

46. If lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-5}$ are perpendicular, the value of k is ....

47. The distance between the parallel lines $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$ and $\vec{r}=\left(3\hat{i}+3\hat{j}-5\hat{k}\right)+\mu\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$ is ....

48. The lines $\frac{x-5}{k}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are perpendicular then k = ...

TWO MARKS QUESTIONS (QUESTION NOS. 21 & 22):

1. Find $\left|\vec{x}\right|$, if for a unit vector $\vec{a}$, $\left(\vec{x}-\vec{a}\right)\cdot\left(\vec{x}+\vec{a}\right)=12$.

2. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a}=3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$.

3. Obtain the projection of the vector $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on the vector $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$.

4. Find $\left|\vec{b}\right|$, if $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=8$ and $\left|\vec{a}\right|=8\left|\vec{b}\right|$.

5. If $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$, prove that $\vec{a}$ and $\vec{b}$ are perpendicular.

6. Find a vector in the direction of the vector $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$ that has magnitude 7 units.

7. If $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}+5\hat{k}$, then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular.

8. Show that $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)$.

9. If either $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$ then $\vec{a}\cdot\vec{b}=0$, but the converse need not be true. Justify your answer.

10. Show that the vector $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the positive directions of the co-ordinate axes.

11. Find the values of λ and μ if $\left(2\hat{i}+b\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right)=\vec{0}$.

12. Find the angle between the vectors $\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\hat{k}$.

13. Show that the points $A(1, 2, 7)$, $B(2, 6, 3)$ and $C(3, 10, -1)$ are collinear.

14. For given vectors, $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}-\hat{k}$, find the unit vector in the direction of the vector $\vec{a}+\vec{b}$

15. Find the direction cosines of the vector joining the points $A(1,\ 2,\ -3)$ and $B(-1,\ -2,\ 1)$, directed from A to B.

16. Find $|\vec{a}-\vec{b}|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=2,\ |\vec{b}|=3$ and $\vec{a}\cdot\vec{b}=4$.

17. If $\vec{a}=2\hat{i}+2\hat{j}+3k,\ \vec{b}=-\hat{i}+2\hat{j}+k$ and $\vec{c}=3\hat{i}+\hat{j}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c},$ then find the value of λ.

18. If $\vec{a},\ \vec{b},\ \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0},$ find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

19. Find $|\vec{a}\times\vec{b}|,$ if $\vec{a}=\hat{i}-7\hat{j}+7k$ and $\vec{b}=3\hat{i}-2\hat{j}+2k$.

20. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-k$ and $-\hat{i}+\hat{j}+k$ respectively in the ratio $2:1$ (i) internally (ii) externally.

21. Consider two points P and Q with position vectors $\overrightarrow{OP}=3\vec{a}-2\vec{b}$ and $\overrightarrow{OQ}=\vec{a}+\vec{b}$. Find the position vector of a point R which divides the line segment joining P and Q in the ratio $2:1$ (i) internally (ii) externally.

THREE MARKS:

1. If $\vec{a}=2\hat{i}+2\hat{j}+3k,\ \vec{b}=-\hat{i}+2\hat{j}+k$ and $\vec{c}=3\hat{i}+\hat{j}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c},$ then find the value of λ.

2. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ where $\vec{a}=\hat{i}+\hat{j}+k,$ $\vec{b}=\hat{i}+2\hat{j}+3k$.

3. Show that the position vector of a point $P,$ which divides the line joining the points A and B having position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio $m:n$ is $\dfrac{m\,\vec{b}+n\,\vec{a}}{m+n}$.

4. Find the area of a triangle having the points $A(1,\ 1,\ 1),\ B(1,\ 2,\ 3)$ and $C(2,\ 3,\ 1)$ as its vertices using vector method.

5. The vectors $\vec{a},\ \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a},$ if $|\vec{a}|=1,\ |\vec{b}|=4$ and $|\vec{c}|=2$.

6. If the vertices $A,\ B$ and C of a triangle are $(1, 2, 3), (-1,\ 0,\ 0)$ and $(0, 1, 2)$ respectively then ind $\underline{|ABC}$.

7. Let $\vec{a},\ \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=3,\ |\vec{b}|=4,\ |\vec{c}|=5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a}+\vec{b}+\vec{c}|$

8. Show that the vectors $2\hat{i}-\hat{j}+k,\ \hat{i}-3\hat{j}-5k$ and $3\hat{i}-4\hat{j}-4k$ form the vertices of a right angled triangle.

FIVE MARK QUESTIONS:

1. Derive the equation of a line in space passing through a given point and parallel to a vector both in the vector and Cartesian form.