SECOND PUC PREPARATORY - 5 2023-2024 – BRIKS ACADEMY PADMANABHANAGARA

SUBJECT : MATHEMATICS (35)

TIME : 3 Hours 15 Minutes **[Total questions : 52]** **Max. Marks : 80**

Instructions :

1. The question paper has five parts namely A, B, C, D and E. Answer all the Parts.

2. Part A has 15 multiple choice questions, 5 fill in the blank questions.

3. Use the graph sheet for question on linear programming problem in Part E.

PART – A

I. Answer ALL the Multiple Choice Questions: **5 x 1 = 5**

1. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $l_1 R l_2$ if & only if l_1 *is perpendicular to* l_2 $\forall\, l_1, l_2 \in L$. The R is
   1) reflexive 2) symmetric 3) transitive 4) none of these
2. Let A={1,2,3,_____n} & B = {a,b}. then the number of surjective from A to B is
   1) $^nP_2$ 2) $2^n - 2$ 3) $2^n - 1$ 4) none of these
3. The value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$ is _____
   1) π 2) $-\frac{\pi}{2}$ 3) 0 4) none of these
4. For 2X2 matrix, $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{(i+j)^2}{2}$ then A is equal to
   1) $\begin{bmatrix} 1 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$ 2) $\begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 4 \end{bmatrix}$ 3) $\begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$ 4) $\begin{bmatrix} 1 & \frac{9}{2} \\ \frac{9}{2} & 4 \end{bmatrix}$
5. In a square matrix A, sum of product of elements of a row (or) column with their corresponding cofactors is
   1) 0 2) detA 3) |adjA| 4) nothing can be said
6. The derivative of $\cos^{-1} x$ exists in the interval
   1) [-1,1] 2) (-1, 1) 3) R 4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
7. If $y = x^{20}$ then $\frac{d^2y}{dx^2} =$
   1) $20x^{19}$ 2) $20x^{18}$ 3) $380x^{18}$ 4) $380x^{19}$
8. Local maxima of the function g(x) = $x^3 - 3x$ is
   1) 1 2) –1 3) –2 4) 2
9. $\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2}\right) dx =$
   1) $\frac{e^x}{1+x^2} + c$ 2) $e^x tanx + c$ 3) $e^x \tan^{-1} x + c$ 4) $\frac{\tan^{-1} x}{1+x^2} + c$
10. $\int_2^3 \frac{dx}{x^2-1} =$
    1) $\log\frac{3}{2}$ 2) $log\sqrt{2}$ 3) $\log(\sqrt{2} - 1)$ 4) $log2$
11. If $\vec{a}$ is a non-zero vector of magnitude a & λ a non-zero scalar, then $\lambda\vec{a}$ is a unit vector if
    1) λ=1 2) λ= –1 3) a = |λ| 4) a = $\frac{1}{|\lambda|}$
12. If $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$ and θ is angle between $\vec{a}$ & $\vec{b}$ then θ is equal to
    1) 0 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) π
13. The direction cosines of z-axis

1) 0,1,0 2) 0,0,1 3) 1,0,0 4) 0,1,1

14. Corner points of feasible region determined by the following system of linear inequalities $2x+y \le 10$, $x+3y \le 15$ where x, $y \ge 0$ are (0,0) (5,0) (3,4) (0,5). Let Z = px+qy where p, $q \ge 0$. Condition on p & q so that maximum of Z occurs at both (3,4) & (0,5) is
1) p = q 2) p = 2q 3) p = 3q 4) q = 3p

15. If A & B are independent events such that P(A)=0.3 & P(B)=0.6 then P(A & notB) is
1) 0.12 2) 0.18 3) 0.28 4) 0.42

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket () 3 x 1 = 3

16. Value of $\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$ is _____.

17. If |A|=10 & |B|= −1, where A & B are square matrices of same order then |AB| =___.

18. Number of arbitrary constants in general solution of differential equation of fourth order is ____

19. If $\frac{1}{\sqrt{2}}, \frac{1}{2}, k$ represents direction cosines of a line then k= ______.

20. Two cards are drawn randomly without replacement from a pack of 52 playing cards. The probability that both are black cards is ______.

PART – B

Answer any SIX of the following questions: 6 x 2 = 12

21. Find the values of the $tan^{-1}(\sqrt{3})$ - $sec^{-1}(-2)$.

22. Find the equation of the joining $(1, 2)$ and $(3, 6)$ using determinants.

23. Find $\frac{dy}{dx}$, if $xy + y^2 = \tan x + y$.

24. Find the rate of change of the area of a circle with respect to its radius r when r=3cm.

25. Find the maximum and minimum values, if any of the function given by $f(x) = |x|, x \in R$.

26. Evaluate $\int \frac{1-\sin x}{cos^2 x} dx$

27. Evaluate $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$.

28. Find the unit vector in the direction of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$

29. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

30. If $P(E) = \frac{6}{11}$, $P(F) = \frac{5}{11}$ and $P(E \cup F) = \frac{7}{11}$, find (i) $P(E \cap F)$ (ii) $P(E|F)$ (iii) $P(F|E)$.

31. If A and B are two independent events, then the probability of occurrence of atleast one of A and B is given by 1 - $P(A')$. $P(B')$.

PART - C

Answer any SIX of the following questions: 6 x 3 = 18

32. Show that function $f : R \to R$ defined by $f(x) = x^4$ is neither one -one nor onto.

33. Write $tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$, $|x| < a$ in the simplest form.

34. Express the following matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ as the sum of a symmetric & skew-symmetric matrix.

35. Find $\frac{dy}{dx}$, if $x^2+xy+y^2 = 100$.

36. Find $\frac{dy}{dx}$, if $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$.

37. Find the intervals in which the function $f(x) = (x+1)^3(x-3)^3$ is
a) strictly increasing (b) strictly decreasing

38. Find $\int x \log 2x \, dx$

39. Find the particular solution of the differential equation $x(x^2-1)\frac{dy}{dx} = 1$; $y = 0$ when $x = 2$

40. Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}\,|$.

41. Find the area of the triangle where position vectors of A,B and c are $\hat{\imath} - \hat{\jmath} + 2\hat{k}$, $2\hat{\jmath}+\hat{k}$ and $\hat{\jmath} + 3\hat{k}$ respectively.

42. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

PART – D

Answer any FOUR the following questions: **4 x 5 = 20**

43. Consider function f: R → R defined by $f(x)=1+x^2$, show that f is one-one & onto, justify your answer.

44. If $A=\begin{bmatrix}1 & 1 & -1\\ 2 & 0 & 3\\ 3 & -1 & 2\end{bmatrix}$ $B=\begin{bmatrix}1 & 3\\ 0 & 2\\ -1 & 4\end{bmatrix}$ $C=\begin{bmatrix}1 & 2 & 3 & -4\\ 2 & 0 & -2 & 1\end{bmatrix}$ Prove that (AB)C=A(BC)..

45. Solve the system of equations 2x +3y+3z=5, x-2y+z=-4, 3x-y-2z=3 by matrix method.

46. If $y = e^{a\cos^{-1}x}$, $-1 \le x \le 1$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$.

47. Find the integral of $\sqrt{x^2 - a^2}$ w.r.t. x and hence evaluation $\int \sqrt{x^2 - 4x + 6}\,dx$.

48. Find the area of the region bounded by the circle $x^2 + y^2 = 16$ using integration.

49. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

50. Derive the shortest distance between the two skew lines both in vector & cartesian form.

PART – E

Answer the following.

51. Prove that $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)\,dx & if\ f(2a-x) = f(x) \\ 0 & if\ f(2a-x) = -f(x) \end{cases}$ **6**

(OR)

Solve the following problem graphically. Minimize & maximise $z = 3x + 9y$ subject to the constraints $x + 3y \le 60, x + y \ge 20, x \le y, x \ge 0, y \ge 0$.

52. Find the relationship between 'a' and 'b' so that the function 'f' defined by

$f(x) = \begin{cases} ax + 1 & if \quad x \le 3 \\ bx + 3 & if \quad x > 3 \end{cases}$ is continuous at $x = 3$. **4**

(OR)

Show that the matrix $A = \begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$, where I is 2X2 identity matrix & O is 2X2 zero matrix. Using this equation, find A^{-1}.