SECOND PUC PREPARATORY - 4 2023-2024 – BRIKS ACADEMY PADMANAHABHANAGARA

SUBJECT : MATHEMATICS (35)

TIME : 3 Hours 15 Minutes **[Total questions : 52]** **Max. Marks : 80**

Instructions :

1. The question paper has five parts namely A, B, C, D and E. Answer all the Parts.

2. Part A has 15 multiple choice questions, 5 fill in the blank questions.

3. Use the graph sheet for question on linear programming problem in Part E.

PART – A

I. Answer ALL the Multiple Choice Questions: **5 x 1 = 5**

1. Which of the following relation in the set {1, 2, 3} is symmetric but neither reflexive nor transitive

   1) {(1,2) (2,1) (1,1) (1,3)} 2) {(1,2) (2,1)}
   3) {(2,3)} 4) {(1,2) (2,1) (1,1) (2,2)}

2. Let A={1,2,3} B={4,5,6,7} & f={(1,4) (2,5) (3,6)} be a function from A to B. choose the correct answer

   1) f is one-one, onto 2) f is many-one, onto
   3) f is one-one but not onto 4) f is neither one-one nor onto

3. The range of $\operatorname{cosec}^{-1} x$ is

   1) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)-\{0\}$ 2) $(0,\pi)-\left\{\frac{\pi}{2}\right\}$ 3) $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ 4) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$

4. The number of all possible matrices of order 3X3 with each entry 0 or 1 is :

   1) 27 2) 18 3) 81 4) 512

5. If A is a matrix of order 3, such that A(adjA)=10I, then |adjA|=

   1) 10 2) $\frac{1}{10}$ 3) 1 4) 100

6. If y = sin(logx), then $\frac{dy}{dx}$ =

   1) cos(logx) 2) -cos(logx) 3) $\frac{\cos(logx)}{x}$ 4) $\frac{-\cos(logx)}{x}$

7. Number of discontinuity points for f(x) = [x], 0<x,3 is

   1) 0 2) 1 3) 2 4) 4

8. The interval in which f(x) = $2x^2 - 3x$ is increasing is

   1) $\left(\frac{3}{4},\infty\right)$ 2) $\left(-\infty,\frac{3}{4}\right)$ 3) R 4) $\left(0,\frac{3}{4}\right)$

9. $\int \frac{x^3-x^2+x-1}{x-1}dx$ is

   1) $x^2 + x + c$ 2) $\frac{x^3}{3}+x+c$ 3) $\frac{x^3}{3}-x+c$ 4) $\frac{x^3}{3}+\frac{x^2}{2}+c$

10. $\int x^2e^{x^3}\,dx$ equals to

    1) $\frac{e^{x^3}}{3}+c$ 2) $\frac{e^{x^2}}{3}+c$ 3) $\frac{e^{x^3}}{2}+c$ 4) $\frac{e^{x^2}}{2}+c$

11. If $\vec{a}=\hat{\imath}+\hat{\jmath}-2\hat{k}$ then direction cosines of $\vec{a}$ is

    1) $\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},-\frac{2}{\sqrt{6}}$ 2) $\frac{1}{6},\frac{-2}{\sqrt{6}},\frac{1}{\sqrt{6}}$ 3) $\frac{-2}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}$ 4) none of these

12. The value of λ for which the vectors $2\hat{\imath}-3\hat{\jmath}+4\hat{k}$ & $-4\hat{\imath}+\lambda\hat{\jmath}-8\hat{k}$ are collinear is

    1) 3 2) 6 3) -3 4) -6

13. If a line has direction ratios 2, -1, -2 then its direction cosines

    1) $\frac{2}{3},-\frac{1}{3},-\frac{2}{3}$ 2) $\frac{2}{3},\frac{1}{3},-\frac{2}{3}$ 3) $\frac{2}{3},\frac{1}{3},\frac{2}{3}$ 4) $\frac{2}{3},-\frac{1}{3},\frac{2}{3}$

14. Optimal value of objective function is attained at the points

    1) on X-axis 2) on Y-axis 3) corner points 4) none of these

15. A urn contains 10 black & 5 white balls, 2 balls are drawn one after the other without replacement. What is the probability that both drawn balls are black?

1) $\frac{3}{7}$ 2) $\frac{4}{9}$ 3) $\frac{1}{9}$ 4) $\frac{2}{21}$

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket () **3 x 1 = 3**

16. Principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is ____.

17. The value of x in which $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$ is ______.

18. Sum of order & degree of $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$ is ______.

19. Lines $\frac{x-1}{3} = \frac{y-2}{2p} = \frac{z-3}{2}$ & $\frac{x-1}{3p} = \frac{y-1}{1} = \frac{z-6}{5}$ are perpendicular, then p = _____.

20. If A & B are independent events with P(A)=0.3, P(B)=0.4, then P(A∩B) is

PART – B

Answer any SIX of the following questions: **6 x 2 = 12**

21. Prove that $2\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{24}{7}\right)$.

22. Find the equation of the line joining A (1, 3) and B (0, 0) using determinants.

23. Find $\frac{dy}{dx}$, if $ax + by^2 = \cos y$.

24. The radius of a circle is increasing uniformly at the rate of 3cm/s. find the rate at which the area of the circle is increasing when the radius is 10cm.

25. Find the maximum and minimum values, if any of the function given by $f(x) = x^2, x \in [-2,1]$

26. Find $\int \frac{dx}{(x+1)(x+2)}$.

27. Evaluate $\int_0^{\frac{\pi}{2}} \left(sin^2\left(\frac{x}{2}\right) - cos^2\left(\frac{x}{2}\right)\right) dx$.

28. Find the projection of $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$.

29. Find the angle between the pair of lines given by (i) $\vec{r} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ $\vec{r} = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$

30. If $P(E) = \frac{7}{13}$, $P(F) = \frac{9}{13}$ and $P(E \cap F) = \frac{4}{13}$, evaluate $P(E|F)$.

31. If A & B two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ & $P(A \cap B) = \frac{1}{8}$, find P(notA & notB).

PART - C

Answer any SIX of the following questions: **6 x 3 = 18**

32. Show that the relation R in the set A of all the books in a library of a college, given by $R = \{(x, y): x \text{ and } y \text{ have same number of pages}\}$ is an equivalence relation.

33. Write the simplest form of $tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $0 < x < \pi$.

34. Express $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of symmetric & skew-symmetric matrix.

35. Find $\frac{dy}{dx}, if\ x^y + y^x = 1$.

36. Find $\frac{dy}{dx}$, if $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$.

37. Find the intervals in which the function $f(x) = 10 - 6x - 2x^2$ is

a) strictly increasing b) strictly decreasing:

38. Evaluate $\int x\, sin^{-1} x\, dx$.

39. Find the particular solution of the differential equation $cos\left(\frac{dy}{dx}\right) = a\ (a \in R)$; $y = 2$ when $x = 0$.

40. Show that the position vector of the point P, which divides the line joining the points A & B having position vectors $\vec{a}$ & $\vec{b}$ internally in the ration m:n is $\frac{m\vec{b}+n\vec{a}}{m+n}$.

41. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.

42. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

PART – D

Answer any FOUR the following questions: 4 x 5 = 20

43. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$. Is f one – one onto? Justify your answer.

44. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ & $A = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ then compute (A+B) & (B-C). Also verify that A+(B-C)=(A+B)-C.

45. Solve the following system of linear equations by matrix method. $x - y + 2z = 1$, $2y - 3z = 1$, $3x - 2y + 4z = 2$.

46. If $y = Ae^{mx} + Be^{nx}$ show that $y_2 - (m+n)y_1 + mny = 0$.

47. Find the integral of $\sqrt{x^2 + a^2}$ w.r.t x and hence evaluate $\int \sqrt{x^2+4x+6}\ dx$.

48. Find the area of the region bounded by the circle $x^2 + y^2 = a^2$ using integration.

49. Find the general solution of the differential equation $x\frac{dy}{dx} + y - x + x\cot x = 0, x \neq 0$

50. Derive the shortest distance between the two skew lines.

PART – E

Answer the following.

51. Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ hence evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\cos^2 x+\sin^2 x}\,dx$ **6**

(OR)

Minimize and maximize Z=5x+10y

subject to the constraints $x + 2y \leq 120$

$x + y \geq 60$,

$x - 2y \geq 0$

$x \geq 0, y \geq 0$ *by graphical method*.

52. Find the values of k so that the function f is continuous at the point x = π where

$f(x) = \begin{cases} kx+1 & \text{if } x \leq \pi \\ \cos x & \text{if } x > \pi \end{cases}$ **4**

(OR)

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = O$, where I is 2X2 identity matrix & O is 2X2 zero matrix. Using this equation, find A^{-1}.

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